4 th
Polish
Combinatorial
Conference

ABSTRACTS

Będlewo, September 17-21, 2012

http://4pcc.tcs.uj.edu.pl

LIST OF TALKS

Mahdi Amani
University of Pisa, Italy

An ECO method for generating and enumerating AVL trees with n nodes

AVL tree which is an object of research in different fields of data structure and algorithms is defined as a binary search tree such that the height of the left and right subtrees of any internal node differs by at most 1. Generation of different types of trees have many application in study of characterization of the trees and it is a significant typical problem in combinatorics and were studied earlier in the literature, but the problem of generation of AVL trees has been studied only by Liwu Li in [1], he presented a new ranking and unranking algorithms of AVL trees in time complexity of $O(n\log^2 n)$ and $O(n\log^3 n)$ respectively, after a preprocessing step that takes $O(n^2\log n)$ then he used the ranking and unranking algorithms to generate the trees so totally his generation algorithm has time complexity of $O(nlog^3n)$. In this work we use a method (called the ECO method [2]) to present a new ECO algorithm to generate level by level all trees of AVL-trees with n nodes in a minimal change ordering and we prove that our generating algorithm has constant time complexity in the worst case (regardless of input and output time). The basic idea of ECO method is: given a class X of combinatorial objects and a parameter$p$ of X, let us consider the set $X_n = \{x \in X : p(x) = n\}$; we try to define an operator which constructs each object $Y \in X_{n+1}$ from another object $Z \in X_n$ (only one) and finally we get a minimal change ordering with respect to that for generating the trees of each level.

References

[1] Liwu Li, Ranking and Unranking of AVL-trees, SIAM J. of Computing 15 (1986), 1025-1035.

[2] E. Barcucci, A. D. Lungo, E. Pergola, R. Pinzani, EC0:A Methodology for the Enumeration of Combinatorial Objects, Journal 5 (1999), 435-490.

Marcin Anholcer
Poznań University of Economics, Poland

Group Irregularity Strength of Digraphs

We investigate the *group irregularity strength* ($\overrightarrow{s_g}(D)$) of digraphs, i.e. the smallest value of s such that taking any Abelian group $\mathcal{G}$ of order s, there exists a function $f : A(D) \to \mathcal{G}$ such that the differences of weighted in- and out-degrees at every vertex are distinct. We give lower and upper bounds for general digraphs and exact values of $\overrightarrow{s_g}(D)$ for some families of digraphs.

Niranjan Balachandran
Indian Institute of Technology, India

Combinatorial Batch codes and an extremal Hypergraph Problem

joint work with: Srimanta Bhattacharya[1]
[1] Indian Statistical Institute, Kolkata, India

An (m, N, k, n, t)-batch code models the problem of storing m data items into n servers in such a way that any k of the m data items may be retrieved by reading at most t items from each server and the overall storage to be limited to N and we are interested in general in minimizing N, over all m servers. The case $t = 1$ corresponds to the situation where every server stores a subset of the items is called a Combinatorial Batch code (CBC). The problem of determining $m(n, r, k)$, the maximum value of n for which there exists a uniform (m, rn, k, n)-CBC has been the object of interest in recent times and it is known that $m(n, r, k) = O(n^r)$ for fixed r, k. We shall provide an asymptotic improvement of the upper bound on $m(n, r, k)$ for all $2 < r < k - 3$, for fixed $k > 6$.

Olivier Baudon
University of Bordeaux, France

On the structure of arbitrarily partitionable graphs with given connectivity

joint work with: Florent Foucaud[1], Jakub Przybyło[2], Mariusz Woź niak[2]
[1]University of Bordeaux, France, [2]AGH University of Science and Technology, Poland

A graph $G = (V, E)$ is arbitrarily partitionable if for any sequence τ of positive integers adding up to $|V|$, there is a sequence of vertex-disjoint subsets of V whose orders are given by τ, and which induce connected subgraphs. Such graph models, e.g., a computer network which may be arbitrarily partitioned into connected subnetworks. In this talk, we study the structure of such graphs and prove that unlike in some related problems, arbitrarily partitionable graphs may have arbitrarily many components after removing a cutset of a given size $\tau \geq 2$. The sizes of these components grow exponentially, though.

Małgorzata Bednarska-Bzdęga
Adam Mickiewicz University, Poland

On the relationship between Picker-Chooser and Maker-Breaker games

Two players claim in turns previously unselected vertices of the hypergraph H. One of the players, let us call him the builder, wins when he claims all elements in at least one edge of the hypergraph; otherwise the other player, the spoiler, is the winner.

In a well-known Maker-Breaker version of the game, Maker, who is the builder, selects p elements and Breaker claims q elements per turn. In the Picker-Chooser version Picker is the builder and Chooser is the spoiler. At every turn Picker selects p+q unoccupied vertices. Chooser keeps q of them and the remaining p elements go to Picker.

We will talk on the relationship between the two versions of the games, a conjecture of Beck, and weight function based winning criteria for Maker/Breaker and Picker/Chooser.

Walid Ben-Ameur
Telecom SudParis, France

On graphs with unicyclic connected components

Given a weighted undirected graph, we want to partition the set of vertices into some connected components such that each connected component contains exactly one cycle. The objective function is given by the weight of the edges included in the solution.

First, we prove that this problem is easy to solve. Then, we add a new technical constraint related to the size of cycles: the solution should not contain cycles of length less than a certain bound. This constraint makes the problem difficult. A polyhedral study is then proposed. Many facets and valid inequalities are derived. Some of them can be exactly separated in polynomial time.

We also consider another important variation of the problem. Some given special nodes must belong to cycles. We still want the connected components to be unicyclic while the cycle size constraint is ignored. We show that this problem is a generalization of the perfect binary 2-matching problem. It turns out that the problem is easy to solve. An exact extended linear formulation is provided. We also present a partial description of the convex hull of the incidence vectors of these Steiner networks. Polynomial time separation algorithms are described. One of them is a generalization of the Padberg-Rao algorithm to separate blossom inequalities.

Julien Bensmail
University of Bordeaux, France

On the size of graphs that can be partitioned under a given number of prescriptions

joint work with: Olivier Baudon[1] Jakub Przybyło[2], Mariusz Woźniak[2], Eric Sopena[1]
[1]University of Bordeaux, France, [2]AGH University of Science and Technology, Poland

A graph G is said to be *arbitrarily partitionable* if for every partition $(\tau_1, ..., \tau_p)$ of $|V(G)|$ there exists a partition $(V_1, ..., V_p)$ of $V(G)$ such that each V_i induces a connected subgraph of G with order τ_i. If, additionally, each of k of these subgraphs contains an arbitrary vertex of G prescribed beforehand, then G is said to be *arbitrarily partitionable under k prescriptions* (AP+k for short). It should be clear that an AP+k graph on n vertices is $(k+1)$-connected and thus has at least $\lceil \frac{n(k+1)}{2} \rceil$ edges. In this talk, we show that there exists an AP+k graph with order n and size $\lceil \frac{n(k+1)}{2} \rceil$ for every $k \geq 1$ and $n \geq k$.

Halina Bielak
Maria Curie-Skłodowska University, Poland

Multicolor Ramsey numbers for some forests and cycles

We give the Turán number for some forests. We extend some known results of Gorgol [5], Bushaw and Kettle [3] for disjoint union of paths, and famous result of Faudree and Schelp [4] for paths. Moreover, applying the result, we give multicolor Ramsey numbers for a sequence of some forests and cycles. We extend some known results of Bielak [1] and Burr and Roberts [2].

References

[1] H. Bielak, Multicolor Ramsey numbers for some paths and cycles, Discussiones Mathematicae Graph Theory 29 (2009), 209-218.

[2] Burr S.A., J.A. Roberts, On Ramsey numbers for linear forests, Discrete Math. 8 (1974), 245-250.

[3] N. Bushaw, N. Kettle, Turán Numbers of Multiple Paths and Equibipartite Forests, Combininatorics, Probability and Computing, 20(6)(2011) 837-853.

[4] R.J. Faudree, R.H. Schelp, Path Ramsey numbers in multicolorngs, J.Combin. Theory Ser. B 19 (1975) 150-160.

[5] I. Gorgol, Turán Numbers for Disjoint Copies of Graphs, Graphs Combin. 27 (2011), 661-667.

Csaba Biro
University of Louisville, USA

The rate of growth of the minimum clique size of graphs of given order and chromatic number

joint work with: Kris Wease[1]
[1] University of Louisville, USA

Let $Q(n,c)$ be the minimum clique number over graphs on n vertices with chromatic number c. In 2009 the speaker determined $Q(n,n-k)$ for small k and large n. In 2011, Biro, Furedi, and Jahanbekam found an exact formula in terms of inverse Ramsey number in case $c > n/2$. In 2012 Liu found the rate of growth of $Q(n,n/b)$ for b fixed integer. In this paper we find the correct rate of growth $Q(n,rn)$ for all $0 < r \leq 1$ fixed real number.

Mindaugas Bloznelis
Vilnius University, Lithuannia

Degree and clustering in sparse intersection graphs

I address two questions. Given integer k, one selects at random a vertex of degree k. What is the probability that two randomly selected neighbours of this vertex are adjacent? Given integer k, one select at random a pair of vertices having k common neighbours. What is the probability that these two selected vertices are adjacent? These questions make sense in real networks where adjacency relations are statistically dependent. I shall present some asymptotic results for sparse random intersection graphs with large number of vertices. A comparison of a real affiliation network data with related intersection graph models will be considered.

Danila Cherkashin
Saint-Petersburg State University, Russia

On the chromatic numbers of small-dimensional spaces

joint work with: A.B. Kulikov[1]
[1] Saint-Petersbourg State University, Russia

We consider the value $\chi(\mathbb{R}^n)$ which is called *the chromatic number of the Euclidean space* and equals the minimum number of colors necessary to paint the points in $\mathbb{R}^n$ in such a way that no two monochromatic points are at the distance 1 apart. The problem of finding $\chi(\mathbb{R}^n)$ is one the most important in combinatorial geometry. However, even $\chi(\mathbb{R}^2)$ is unknown. So many attepmts have been done to provide good lower and upper bounds for the chromatic numbers in small dimensions. In particular, the following results were obtained:

dim	1	2	3	4	5	6	7	8	9	10	11	12
$\chi \geq$	2	4	6	7	9	11	15	16	21	23	25	27

In order to obtain lower bounds, finite distance graphs are used. By a *distance graph* we mean a graph $G = (V, E)$ such that $V \subset \mathbb{R}^n$, $E \subseteq \{\{\mathbf{x}, \mathbf{y}\} : |\mathbf{x} - \mathbf{y}| = a\}$ with some $a > 0$.

In our work, we consider a series of distance graphs $G_n = (V_n, E_n)$, where

$$V_n = \left\{ (v_1, \ldots, v_n) : \ v_i \in \{-1, 0, 1\}, \quad \sum_{i=1}^{n} |v_i| = 3 \right\}, \quad E_n = \{\{\mathbf{x}, \mathbf{y}\} : (\mathbf{x}, \mathbf{y}) = 1\}.$$

We succeeded in making a full classification of the maximal independent sets in the graphs G_n. Thus, we proved the following theorem concerning the independence number $\alpha(G_n)$ of G_n.

Theorem 1. *The following equalities hold:*

$$\text{if } n = 4k, \quad \alpha(G_n) = \max\{4n, 6n - 28\};$$

$$\text{if } n = 4k + 1, \quad \alpha(G_n) = \max\{4n - 4, 6n - 28\};$$

$$\text{if } n = 4k + 2, \quad \alpha(G_n) = \max\{4n - 8, 6n - 28\};$$

$$\text{if } n = 4k+3, \quad \alpha(G_n) = \max\{4n-8, 6n-28\}.$$

Using Theorem 1, some modifications of the graphs G_n, and some computer calculations made by V.V. Sokolov we obtained the following results:

dim	7	8	9	10	11	12
$\chi \geq$ (old result)	15	16	21	23	25	27
$\chi \geq$ (new result)	16	17	22	30	35	37

Dennis Clemens
Freie Universität Berlin, Germany

Fast strategies in Maker-Breaker games played on random boards

joint work with: Asaf Ferber[1], Anita Liebenau[2], Michael Krivelevich[1]
[1]Tel Aviv University, Israel, [2]Freie Universität Berlin, Germany

A Maker-Breaker game is defined as follows: Given a board X and a set of winning sets $F \subset 2^X$, two players, called Maker and Breaker, alternately take elements of X. If Maker occupies an element of F completely until the end of the game, he wins. Otherwise Breaker is the winner.

We will see that, playing on the edge set of a sparse random graph $G \sim G_{n,p}$ with $p = (\log n)^d/n$ (d large enough), Maker can claim a perfect matching , a Hamiltonian cycle or a k-connected spanning subgraph as fast as possible, i.e., in $n/2 + o(n)$, $n + o(n)$ and $kn/2 + o(n)$ moves, respectively.

Artur Czumaj
University of Warwick, UK

Fast algorithms for finding cycles and trees in bounded degree graphs

joint work with: Oded Goldreich[1], Dana Ron[2], C. Seshadhri[3], Asaf Shapira[4], Christian Sohler[5]
[1]Weizmann Institute of Science, Israel, [2]Tel Aviv University, Israel, [3]Chennai Mathematical Institute, India, [4]Georgia Institute of Technology, USA [5]Technische Universität Dortmund, Germany

We will present sublinear-time (randomized) algorithms for finding simple cycles of length at least $k \geq 3$ and tree-minors in bounded-degree graphs. The complexity of these algorithms is related to the distance of the graph from being C_k-minor free (resp., free from having the corresponding tree-minor). In particular, if a constant fraction of the edges must be deleted to make the graph cycle-free, then the algorithm finds a cycle of polylogarithmic length in time $O{*}(\sqrt{N})$, where N denotes the number of vertices. This time complexity is optimal up to polylogarithmic factors. These results are obtained as the outcome of our study of the complexity of one-sided error property testing algorithms in the bounded-degree graphs model.

Jean-Marie Droz
Universität Bremen, Germany

Homotopy theories for a category of graphs

Quillen model structures are a way of formalising homotopy theories. Since the definition of a model structures is expressed in the language of category theory, it can be applied to categories of graphs. We will show that classical graph theoretical notions, like the core of a graph and the set of connected components, are notions of homotopy types in specific Quillen model structures. We also explain some negative results proving that what could be regarded as natural candidates for homotopy types in the category of graphs cannot arise from a model structure. Finally, we count the number of model structures on the category of graphs.

Tomasz Dzido
University of Gdańsk, Poland

On some on-line Ramsey numbers

joint work with: Joanna Cyman[1]
[1]Gdansk University of Technology, Poland

We consider on-line Ramsey numbers defined by a game played between two players, Builder and Painter. In one round Builder draws an edge and Painter colors it either red or blue, as each appears. The goal of Builder is to force Painter to create a monochromatic copy of a fixed graph H in as few rounds as possible. The minimum number of rounds (assuming both players play perfectly) is the *on-line Ramsey number* $\widetilde{r}(H)$ of the graph H. An asymmetric version of the on-line Ramsey numbers $\widetilde{r}(G,H)$ is defined accordingly. In this area of small on-line Ramsey numbers, very little is known. In talk we present known and some new results.

Anna Fiedorowicz
University of Zielona Góra, Poland

Acyclic improper colourings of graphs with fixed maximum degree

joint work with: Elżbieta Sidorowicz[1]
[1]University of Zielona Góra, Poland

We consider a vertex k-colouring of a graph G, which can be defined as a mapping c from the set of vertices of G to the set $\{1, \ldots, k\}$ of colours. One can also regard a k-colouring of G as a partition of the set $V(G)$ into colour classes $V_1, \ldots, V_k$ such that each V_i is the set of vertices coloured with i. In many situations it is desired that the particular set V_i has some property, for instance, is independent or induces a subgraph with bounded maximum degree. In the former case it yields to a proper k-colouring, in the latter to a t-improper k-colouring. One can also required that for any pair of distinct colours i and j the subgraph induced by the edges whose endpoints have colours i and j satisfies a given property, for instance, is acyclic. This yields to the concept of acyclic colourings, introduced by Grünbaum in 1973. We mainly concentrate on colourings which are both t-improper and acyclic.

Let $P_1, \ldots, P_k$ be nonempty classes of graphs, closed with respect to isomorphism. A k-colouring of a graph G is called a $(P_1, \ldots, P_k)$-colouring of G if for any $i \in \{1, \ldots, k\}$ the subgraph induced in G by the colour class V_i belongs to P_i. Such a colouring is called an acyclic $(P_1, \ldots, P_k)$-colouring of G, if for every two distinct colours i and j the subgraph induced by the edges whose endpoints have colours i and j is acyclic. An acyclic $(P_1, \ldots, P_k)$-colouring of G such that for any $i \in \{1, \ldots, k\}$ the class P_i is the set of graphs with maximum degree at most t is called an acyclic t-improper k-colouring. The acyclic t-improper chromatic number of a graph G is the smallest k for which there exists an acyclic t-improper k-colouring of G.

We consider acyclic $(P_1, \ldots, P_k)$-colourings of graphs with fixed maximum degree. In particular, we give linear-time algorithms for acyclic t-improper colourings of graphs with maximum degree d. Next, we study acyclic t-improper colourings of graphs with maximum degree 4 and 5 and we determine the upper bounds for the acyclic $(d-1)$-improper chromatic number of graphs with maximum degree d, for $d = 4, 5$.

Radoslav Fulek
Ecole Polytechnique Fédérale de Lausanne, Switzerland

Hanani-Tutte for Two Clusters

joint work with: Jan Kyncl[1], Dömötör Pálvölgyi[2]
[1]Charles University, Prague, Czech Republic, [2]Eötvös Loránd University, Hungary

The classical Hanani-Tutte Theorem says that a graph is planar if it can be drawn in the plane so that no pair of non-adjacent edges cross an odd number of times. We prove a strengthening of the theorem in which vertices are partitioned into two sets A and B; the vertices of A are drawn inside a disc D, vertices of B outside of D, and no edge can intersect the boundary of D more than once.

Adam Gągol
Marie-Curie Skłodowska University, Poland

Structure of Weighted Graphs with Forbidden Subdivision and Graph Sharing Games

joint work with: Piotr Micek[1], Bartosz Walczak[1]
[1]Jagiellonian University, Poland

We show that every weighted graph excluding subdivision of a fixed (small) graph contains one of the following structures:

- a connected subgraph separating the graph into heavy balanced components,
- a heavy set of vertices connected by paths as in a cycle.

This characterization yields a strategy for the first player to gather a positive fraction of the total weight in a variant of graph sharing game played on graphs with forbidden subdivision.

Roman Glebov
Freie Universität Berlin, Germany

On bounded degree spanning trees in the random graph process

joint work with: Daniel Johannsen[1], Michael Krivelevich[1]
[1]Tel Aviv University, Israel

The appearance of certain spanning subgraphs in the random graph is a well-studied phenomenon in probabilistic graph theory. In this talk, we present results on the threshold for the appearance of bounded-degree spanning trees in $G(n,p)$ as well as for the corresponding universality statements. In particular, we show hitting time thresholds for some classes of bounded degree spanning trees.

Vitaly Goldshteyn
Moscow Institute of Physics and Technology, Russia

The Borsuk and Grünbaum problems for $(0,1)$- and $(-1,0,1)$-polyhedra in low dimensions

joint work with: A.M. Raigorodskii[1]
[1]Moscow State University

Our work is motivated by two classical problems — Borsuk's problem on partitioning sets into parts of smaller diameters and Grünbaum's problem on covering sets by balls of the same diameters.

In the case of Borsuk' problem, we study the so-called graphs of diameters. A graph $G = (V, E)$ is said to be *a graph of diameters* for the (finite) set V, if $V \subset \mathbb{R}^n$ and $E = \{\{\mathbf{x}, \mathbf{y}\} : |\mathbf{x} - \mathbf{y}| = \mathrm{diam}\, V\}$. Here we consider only the cases $V \subset \{0,1\}^n$ and $V \subset \{-1,0,1\}^n$. We get the folowing results for the chromatic numbers $\chi(G)$ of such graphs:

- $\chi(G) \leq n + 1$ for any $n \leq 9$ and $V \subset \{0,1\}^n$;
- $\chi(G) \leq n + 1$ for any $n \leq 6$ and $V \subset \{-1,0,1\}^n$.

The first result has already been proved by G.M. Ziegler et al. We reproduce it using a completely different method. An extension of this method allows us to prove the second result, which is new.

In the case of Grünbaum's problem, we study the minimum numbers of balls of the same diameter needed to cover an arbitrary set $V \subset \{0,1\}^n$ of that diameter or an arbitrary set $V \subset \{-1,0,1\}^n$ of that diameter. More precisely, let $\bar{C}_d$ be a closed ball of diameter d and let C_d be an open ball of diameter d. Furthermore, let $\bar{g}(V)$ be the minimum number of balls $\bar{C}_{\mathrm{diam}\, V}$ needed to cover V and let $g(V)$ be the minimum number of balls $C_{\mathrm{diam}\, V}$ needed to cover V. We get the following results:

- $\bar{g}(V) \leq n + 1$ for any $n \leq 9$ and $V \subseteq \{0,1\}^n$;
- $g(V) \leq n + 1$ for any $n \leq 7$ and $V \subseteq \{0,1\}^n$;
- $\bar{g}(V) \leq n + 1$ for any $n \leq 5$ and $V \subseteq \{-1,0,1\}^n$;
- $g(V) \leq n + 1$ for any $n \leq 4$ and $V \subseteq \{-1,0,1\}^n$.

Przemysław Gordinowicz
Technical University of Łódź, Poland

A new approach to calculating small on-line Ramsey numbers for cliques

joint work with: Paweł Prałat[1]
[1]Ryerson University, Canada

We consider an on-line variant of size-Ramsey numbers introduced independently by Kurek and Ruciński, and Beck. The game between two players, Builder and Painter, is played on an infinite set of vertices. In each turn Builder presents one new edge which is immediately coloured red or blue by Painter. The goal of Builder is to force Painter to create either a red copy of G or a blue copy of H for some (fixed in advance) graphs G and H. The goal of Painter is to avoid it for as long as possible. The least number of edges which Builder must draw playing against the perfect opponent is called the on-line Ramsey number $\overline{\mathcal{R}}(G, H)$ for G and H. For simplicity we use $\overline{\mathcal{R}}(k, l)$ to denote $\overline{\mathcal{R}}(K_k, K_l)$.

In the most interesting case of the two graphs being cliques, only two nontrivial numbers are known, namely $\overline{\mathcal{R}}(3, 3) = 8$ and $\overline{\mathcal{R}}(3, 4) = 17$. The lower bound of $\overline{\mathcal{R}}(3, 4)$ was computed by Prałat using computer support, and it took roughly one million CPU hours to verify the bound.

We introduce a new approach to the problem which improves dramatically the computation time (to roughly 1.5 CPU hours for the mentioned bound of $\overline{\mathcal{R}}(3, 4)$). Using this new tool we were able to establish a few new lower bounds for $\overline{\mathcal{R}}(3, 5)$, $\overline{\mathcal{R}}(4, 4)$ and $\overline{\mathcal{R}}(3, 3, 3)$ (the last one being an obvious generalization to three colours).

Mariusz Grech
University of Wrocław, Poland

Černý

joint work with: Andrzej Kisielewicz[1]
[1]University of Wrocław, Poland

The most famous and longstanding problem in the theory of automata is Černý conjecture.

An automaton is resetting if there is a word in the alphabet, such that the image consists of only one state.

Conjecture: For every automaton A on n states, if A is resetting then there exists a word that resets A and has the length at most $(n-1)^2$.

We present a solution of the conjecture for a new class of automata, so cold two-border automata.

Definition: Let A be a finite automaton on a set of states X. Let $\{f_1, \dots, f_k\}$ be the set of function. For every function f_i we define a graph $G_i = (X, E_i)$, where $(x, y) \in E_i$ if and only if $f_i(x) = y$. Let $G = (X, \bigcup E_i)$. Let X_i be the set of these elements of X which belong to some nontrivial component of G_i. We say that A is *two-border* if $\sum_{j \neq i} |X_i \cap X_j| \leq 2$.

Harald Gropp
Universität Heidelberg, Germany

Configurations as graphs and hypergraphs

Configurations are linear regular uniform hypergraphs. On the other hand, they are closely related to regular bipartite graphs. After a short introduction on the history of configuration since 1876 the current state of knowledge will be discussed in both languages, in the language of configurations as well as in the language of graphs and hypergraphs. If time allows, also lambda-configurations will be introduced. At the end, some open problems will be exhibited.

Codrut Grosu
Freie Universität Berlin, Germany

On the rank of higher inclusion matrices

joint work with: Yury Person[1], Tibor Szabó[1],
[1]Freie Universität Berlin, Germany

Let $r \geq s \geq 0$ and G be an r-graph. The higher inclusion matrix $M_s^r(G)$ is a $\{0,1\}$-matrix with rows indexed by edges of G and columns indexed by subsets of $V(G)$ of size s: the entry corresponding to an edge e and a subset S is 1 if $S \subset e$ and 0 otherwise. In this talk I will start by discussing the problem of finding an algebraic analogue of the Kruskal-Katona theorem and how inclusion matrices fit into this picture. Motivated by a question of Frankl and Tokushige and a result of Peter Keevash, I will then define the rank-extremal function $\mathrm{rex}(n, t, r, s)$ as the maximum number of edges of an r-graph G having $\operatorname{rk} M_s^r(G) \leq \binom{n}{s} - t$. For t at most linear in n we have determined this function as well as the extremal r-graphs. I will explain how this result implies the Kruskal-Katona theorem for a certain (but rather modest) range of parameters and how the special case $t = 1$ answers a question of Peter Keevash. As time permits, I will give insights into the proof.

Katarzyna Jesse-Józefczyk
University of Zielona Góra, Poland

Global security in graphs

joint work with: Elżbieta Sidorowicz[1],
[1]University of Zielona Góra, Poland

The talk concerns global secure sets in graphs. A global secure set is a dominating set D such that every its subset X has at least as many neighbours inside D as it has in V-D, where V denotes the set of vertices of a graph. We present some new results on the properties of global secure sets.

Katharina Jochemko
Freie Universität Berlin, Germany

Arithmetic of marked order polytopes, monotone triangle reciprocity, and partial colorings

joint work with: Raman Sanyal[1],
[1]Freie Universität Berlin, Germany,

A map f from a poset P into the n-chain is order preserving if $f(p) \leq f(q)$ whenever $p < q$ in P and equality is prohibited for strict order preservation. For a finite poset P, Stanley considered the problem of counting (strictly) order preserving maps from P into the n-chain and showed that many problems in combinatorics such as counting graph colorings can be cast into this form. He showed that the number of order preserving maps into a chain of length n is given by a polynomial in the positive integer n and gave an interpretation for evaluating this polynomial at negative integers in terms of strictly order preserving maps. This is a special case of the following more general problem: Given a finite poset P, a sub-poset A which contains all minimal and maximal elements of P, and an order preserving map f from A into the integers. What is the number of integral valued order preserving maps with domain P extending f? Such extensions naturally correspond to lattice points in certain polytopes parametrized by f. Using this geometric perspective, we show that the function counting integral valued extensions is a piecewise polynomial in the values of f and we give an interpretation for evaluation at order reversing maps. We use these results to give a geometric proof of a combinatorial reciprocity for monotone triangles due to Fischer and Riegler (2011) and we extend results of Herzberg and Murty (2007) regarding completions of partial colorings of graphs.

Konstanty Junosza-Szaniawski
Warsaw University of Technology, Poland

Determining the $L(2,1)$-Span in Polynomial Space

joint work with: Jan Kratochvil[1], Mathieu Liedloff[2], Paweł Rzążewski[3]
[1] Charles University, Prague, Czech Republic, [2]Laboratoire d'Informatique Fondamentale d'Orleans, France, [3]Warsaw University of Technology, Poland

An $L(2,1)$-labeling of a graph is a mapping from its vertex set into a set of integers $\{0,..,k\}$ such that adjacent vertices get labels that differ by at least 2 and vertices in distance 2 get different labels.

The main result of the paper is an algorithm finding an optimal $L(2,1)$-labeling of a graph (i.e. an $L(2,1)$-labeling in which the largest label is the least possible) in time $O^*(complexity^n)$ and polynomial space. We also adapt our algorithm to get a better time complexity for a $k-L(2,1)$-labeling problem (for small values of k), i.e. verifying if the labels from 0 to k are sufficient to label a given graph.

Moreover, a new interesting extremal graph theoretic problem is defined and solved.

Hrant Khachatrian
Yerevan State University, Armenia

Interval total colorings of complete multipartite graphs

joint work with: Petros Petrosyan[1],
[1]Yerevan State University, Yerevan, Armenia

A total coloring of a graph G is a coloring of its vertices and edges such that no adjacent vertices, edges, and no incident vertices and edges obtain the same color. An interval total t-coloring of a graph G is a total coloring of G with colors $1, \dots, t$ such that at least one vertex or edge of G is colored by i, $i = 1, \dots, t$, and the edges incident to each vertex v together with v are colored by $d_G(v) + 1$ consecutive colors, where $d_G(v)$ is the degree of the vertex v in G. A graph G is interval total colorable if it has an interval total t-coloring for some positive integer t. For an interval total colorable graph G, the least and the greatest values of t for which G has an interval total t-coloring are denoted by $w_\tau(G)$ and $W_\tau(G)$, respectively. In this talk we present some bounds for $w_\tau\left(K_{n_1,\dots,n_k}\right)$ and $W_\tau\left(K_{n_1,\dots,n_k}\right)$ of the interval total colorable complete k-partite graph $K_{n_1,\dots,n_k}$. We also formulate some open problems on interval total colorings of complete multipartite graphs.

Sebastian Kieliszek
Maria Curie-Skłodowska University, Poland

Turan numbers for forests with small components by Sebastian Kieliszek

joint work with: Halina Bielak[1],
[1]Maria Curie-Skłodowska University, Poland

The Turan number $ex(n;G)$ of a graph G is the maximum number of edges in a graph on n vertices which does not contain G as a subgraph. Let P_l denote a path on l vertices and kP_l denote k disjoint copies of P_l. Bushaw and Kettle [1] determined $ex(n;P_l)$ for arbitrary l and n appropriately large relative to k and l. They generalized results of Gorgol [2]. We give a construction to solving the Turan problem for $2P_4$; $3P_4$ and $2P_5$ for all n. Moreover, we consider the problem $ex(n;L)$, where L is a forest with small components.

References

[1] N. Bushaw, N. Kettle, Turan Numbers of Multiple Paths and Equibipartite Forests, Combininatorics, Probability and Computing, 20(6)(2011) 837-853.

[2] I. Gorgol, Turan Numbers for Disjoint Copies of Graphs, Graphs Combin. 27 (2011) 661-667.

Andrzej Kisielewicz
University of Wrocław, Poland

A progress in the Černý conjecture

joint work with: Mariusz Grech[1],
[1]University of Wrocław, Poland

The Černý's conjecture states that for every synchronizing automaton with n states there exists a reset word of length not exceeding $(n-1)^2$. We prove this conjecture for a class of automata preserving certain properties of intervals of a directed graph. Our results unify and generalize some earlier results obtained by other authors.

Fiachra Knox
University of Birmingham, UK

Edge-disjoint Hamilton cycles in random graphs

joint work with: Daniela Kühn[1], Deryk Osthus[1],
[1]Birmingham University, UK

For a graph G on n vertices, call a matching M in G *optimal* if $|M| = \lfloor n/2 \rfloor$. We say that a graph G has *property* $\mathcal{H}$ if G contains $\lfloor \delta(G)/2 \rfloor$ edge-disjoint Hamilton cycles, together with an additional edge-disjoint optimal matching if $\delta(G_0)$ is odd. Not many graphs are known which have property $\mathcal{H}$ (one well-known example is K_n).

Frieze and Krivelevich conjectured that with high probabilty (whp) the binomial random graph $G_{n,p}$ has property $\mathcal{H}$ for any $p = p(n)$. Partial results towards this conjecture have been proved by several authors, including Ben-Shimon, Bollobás, Frieze, Krivelevich, Samotij and Sudakov.

The main result I will present in this talk is to show that this conjecture holds as long as p is not too small and not too large. More precisely, let $\log^{50} n/n \le p \le 1 - n^{-23/64}$. Then whp, $G_{n,p}$ has property $\mathcal{H}$.

Recently, Krivelevich and Samotij covered the range $\log n/n \le p \le n^{-1+\varepsilon}$ and Kühn and Osthus covered the range $p \ge 2/3$. Finally, a well-known result of Bollobás and Frieze covers the range $p \le \log n/n$. So together, these four results prove the conjecture.

We deduce our result from a purely deterministic result which states that every graph which satisfies certain pseudorandomness conditions and which is close (but not too close) to being regular has property $\mathcal{H}$.

Andrei Kokotkin
Moscow State University, Russia

On the realization of random graphs by graphs of diameters

joint work with: Andrei Raigorodskii[1],
[1]Lomonosov Moscow State University, Russia

Our work is motivated by the classical Borsuk problem on partitioning sets into parts of smaller diameters. We study the structure of the so-called graphs of diameters. A graph $G = (V, E)$ is said to be *a graph of diameters* for the (finite) set V, if $V \subset \mathbb{R}^n$ and $E = \{\{\mathbf{x}, \mathbf{y}\} : \ |\mathbf{x} - \mathbf{y}| = \operatorname{diam} V\}$.

Let $G(n, p)$ be a random graph in the Erdős–Rényi model. We are interested in knowing how large can be its induced subgraph that, with high probability, can be realized as a graph of diameters.

Here we consider only the case of $\mathbb{R}^2$. It is well-known (due to Borsuk and Erdős) that for any graph $H \subset \mathbb{R}^2$ of diameters, its chromatic number $\chi(H)$ does not exceed 3. Put

$$u(n,p) = \max \{k : \ \mathrm{P}\,(\exists\ H = (W,F) \subset G(n,p) :$$

$$|W| = k,\ H = G(n,p)|_W,\ H - \text{graph of diameters},\ \chi(H) = 3) > 1/2\}\,.$$

If for any k,

$$\mathrm{P}\,(\exists\ H = (W,F) \subset G(n,p) : \ |W| = k,\ H = G(n,p)|_W,$$

$$H - \text{graph of diameters},\ \chi(H) = 3) \le 1/2,$$

then we put $u(n,p) = 0$.

In our talk, we shall first give an almost exhaustive list of bounds for $u(n,p)$ depending on the edge probability p. Then we shall speak about some multidimensional generalizations.

Daniel Kotlar
Tel-Hai College, Israel

On Extensions of the Alon-Tarsi Latin Square Conjecture

The Alon-Tarsi Latin square conjecture states that for even dimensions, the numbers of even and odd Latin squares are different. There are two known ways to extend this conjecture to odd dimensions. It is shown how these extensions are related to expressions containing a product of the $(n-1)$th power of the determinant with the permanent of certain matrices. One of these expressions will relate the corresponding extended conjecture to a weak case of G-C Rota's base conjecture for odd dimensions.

Jakub Kozik
Jagiellonian University, Poland

ENTROPY COMPRESSION FOR GRAPH COLOURING PROBLEMS

Recent constructive proof of Lovasz Local Lemma, by Moser and Tardos (2009), made it possible to reinterpret applications of the lemma in terms of some specific double counting. An approach based on this idea, originally applied to nonrepetitive sequences by Grytczuk, Kozik and Micek (2011), has been recently extended to some graph colouring problems. I will discuss recent results obtained by applying this method to nonrepetitive vertex colourings and acyclic edge colourings.

Dieter Kratsch
University of Lorraine, France

Exact exponential algorithms

Today most computer scientists believe that NP-hard problems cannot be solved by polynomial time algorithms. While from the polynomial-time perspective all NP-complete problems are equivalent, their exponential-time properties vary widely. Why do some NP-hard problems seem to be easier than others? What are the algorithmic techniques for solving hard problems significantly faster than by the use of exhaustive search, e.g. by trying all possible solutions? Algorithms addressing these questions are known as exact exponential algorithms.

The history of exact exponential algorithms for NP-hard problems dates back at least to the 1960s. Two classical examples are Bellman, Held and Karp's dynamic programming algorithm for the Traveling Salesman problem and Ryser's inclusion-exclusion formula for the permanent. The design and analysis of exact algorithms leads to a better understanding of hard problems and initiates interesting new combinatorial and algorithmic challenges.

The last decade has witnessed a rapid development of the area. This has transformed exact exponential algorithms into an active research field. This talk provides an introduction to exact exponential algorithms and describes various of its fundamental algorithmic techniques.

Tomasz Krawczyk
Jagiellonian University, Poland

Extending partial representations of function graphs and permutation graphs

joint work with: Pavel Klavik[1], Jan Kratochvil[1], Bartosz Walczak[2]
[1]Charles University, Prague, Czech Republic, [2]Jagiellonian University, Poland

Function graphs are graphs representable by intersections of continuous real-valued functions on the interval $[0, 1]$ and are known to be exactly the complements of comparability graphs. As such they are recognizable in polynomial time. Function graphs generalize permutation graphs, which arise when all functions considered are linear.

We focus on the problem of extending partial representations, which generalizes the recognition problem. We observe that for permutation graphs an easy extension of Golumbic's comparability graph recognition algorithm can be exploited. This approach fails for function graphs. Nevertheless, we present a polynomial-time algorithm for extending a partial representation of a graph by functions defined on the entire interval $[0, 1]$ provided for some of the vertices. On the other hand, we show that if a partial representation consists of functions defined on subintervals of $[0, 1]$, then the problem of extending this representation to functions on the entire interval $[0, 1]$ becomes NP-complete.

Przemysław Krysztowiak
Nicolaus Copernicus University, Poland

Recent progress on the jump number of interval orders

The jump number problem is to find a linear extension of a finite poset in which the number of incomparable adjacent pairs is minimized. Its NP-completeness has been proved for bipartite orders [6] and for interval orders [5]. In recent years, a contribution has been made to the algorithms for this problem on interval orders. The algorithms of Felsner [1], Syslo [7], and Mitas [5] have been compared with each other [3], and the Mitas ideas have been extended to yield improved approximation [4] and exact [2] algorithms. The objective of the talk is to review the latest results in the area of algorithms for the jump number problem.

References

[1] S. Felsner, A 3/2-approximation algorithm for the jump number of interval orders, Order 6 (1990), 325-334.

[2] P. Krysztowiak, Exact computation of the jump number on interval orders, January 2012, under review in Discussiones Mathematicae Graph Theory.

[3] P. Krysztowiak, M.M. Syslo, An experimental study of approximation algorithms for the jump number problem on interval orders, February 2012, under review in Discrete Applied Mathematics.

[4] P. Krysztowiak, An improved approximation ratio for the jump number problem on interval orders, February 2012, under review in Theoretical Computer Science.

[5] J. Mitas, Tackling the jump number of interval orders, Order 8 (1991), 115–132.

[6] W.R. Pulleyblank, On minimizing setups in precedence-constrained scheduling, Report No. 81185 - OR (May 1981), unpublished.

[7] M.M. Syslo, The jump number problem on interval orders: A 3/2 approximation algorithm, Discrete Mathematics 144 (1995), 119–130.

Krzysztof Krzywdziński
Adam Mickiewicz University, Poland

A new problem generalising the Ramsey number

The problem is to estimate the number $g(k,l)$. Given positive integers k and l, $g(k,l)$ is the minimum number n such that for any family graphs $G_1, G_2, \ldots G_l$ on n vertices, there are two graphs G_i, G_j, $i \neq j$ such that G_i contains induced subgraph of size k isomorphic to some induced subgraph of G_j. We prove that $g(k,3) = R(k,k)$. We also show some upper and lower bounds for $l > 3$.

Andrey Kupavskii
Moscow State University, Russia

On some geometric Ramsey theory problem

joint work with: Maria Titova[1]
[1]Moscow State University, Russia

We study a problem that connects the classical Ramsey theory and the notion of distance graphs. A *unit distance graph in the n-dimensional Euclidean space* is a graph $G = (V, E)$, whose set of vertices V is a subset of $\mathbb{R}^n$ and

$$E = \{(\mathbf{x}, \mathbf{y}) : \ \mathbf{x}, \mathbf{y} \in V, \ \ |\mathbf{x} - \mathbf{y}| = 1\}.$$

The study of distance graphs is motivated by the well-known problem of finding the chromatic number of a space.

Distance Ramsey number $R_{\mathrm{NEH}}(s, t, n)$ is the minimum $m \in \mathbb{N}$ such that for any graph G on m vertices either G contains an induced s-vertex subgraph isomorphic to a distance graph in $\mathbb{R}^n$ or its complement $\bar{G}$ contains an induced t-vertex subgraph isomorphic to a distance graph in $\mathbb{R}^n$.

It is easy to see that for any n we have $R_{\mathrm{NEH}}(s, t, n) \le R(s, t)$, where $R(s, t)$ is the classical Ramsey number. This inequality holds since an empty graph of any size can be realized as a distance graph in $\mathbb{R}^n$ for $n > 0$.

Lower bounds on $R_{\mathrm{NEH}}(s, s, n)$, as well as on $R(s, s)$, are based on the analysis of the properties of the random graph $G(n, 1/2)$. To find lower bounds on $R_{\mathrm{NEH}}(s, t, n)$ for fixed n one has to utilize some information about all distance graphs in $\mathbb{R}^n$.

For example, the best known lower bounds on $R_{\mathrm{NEH}}(s, s, 2), R_{\mathrm{NEH}}(s, s, 3)$ are obtained as follows. On the one hand, we use the fact that for any distance graph G on s vertices in $\mathbb{R}^2$ or $\mathbb{R}^3$ the number of edges in G is $o(s^2)$ as s grows. On the other hand, if m is chosen properly then with positive probability any subgraph on s vertices in $G(m, 1/2)$ and its complement has $\Omega(s^2)$ edges. So there exists a graph G on m vertices such that neither G nor $\bar{G}$ contains any s-vertex distance graph in $\mathbb{R}^2$ ($\mathbb{R}^3$) as a subgraph.

We derive new lower bounds on the value $R_{\mathrm{NEH}}(s, t, n)$ for small n. We use two approaches.

The first one is based on the independence numbers of distance graphs in $\mathbb{R}^n$. To obtain new bounds we construct dense sets that avoid unit distance in the corresponding spaces.

The second approach generalizes the "edge approach" that gives the best results for $n = 2, 3$. We prove that any distance graph on s vertices in $\mathbb{R}^n$ contains $o(s^d)$ cliques of size d for $d \geq \left[\frac{n}{2}\right] + 1$. We also develop a technique to bound the probability of the event $\{G(m, 1/2)$ has a subgraph on s vertices with $o(s^d)$ d-cliques$\}$.

Valentas Kurauskas
Vilnius University, Lithuannia

Largest clique in sparse random intersection graphs

We study the asymptotic order of the largest clique in sparse uniform random intersection graphs.

John Lapinskas
University of Birmingham, UK

Optimal packings of edge-disjoint Hamilton cycles in graphs of high minimum degree

joint work with: Daniela Kühn[1], Deryk Osthus[1],
[1]Birmingham University, UK

We study the number of edge-disjoint Hamilton cycles one can guarantee in a sufficiently large graph G on n vertices with minimum degree $d = (1/2 + a)n$. For any constant $a > 0$, we give an optimal answer in the following sense: let $\text{reg}_{even}(n, d)$ denote the degree of the largest even-regular spanning subgraph one can guarantee in a graph G on n vertices with minimum degree d. Then we find $\text{reg}_{even}(n, d)/2$ edge-disjoint Hamilton cycles. The value of $\text{reg}_{even}(n, d)$ is known for infinitely many values of n and d. We also extend our results to graphs G of minimum degree $d \geq n/2$, unless G is close to the extremal constructions for Dirac's theorem. Our proof relies on a recent and very general result of Kühn and Osthus on Hamilton decomposition of robustly expanding regular graphs.

Michał Lasoń
Jagiellonian University, Poland

Splitting multidimensional necklace problem

The famous "splitting necklace theorem" of Alon asserts that any (discrete) 1-dimensional k-colored necklace with number of beads of each color divisible by q can be fairly split between q ŤthievesŤ using at most k(q - 1) cuts. It was deduced from a continuous version of the problem. De Longueville and Zivaljevic generalized this version to arbitrary dimension. A (continuous) d-dimensional measurably k-colored necklace is a cube inR^d partitioned into k LebesguŠe measurable sets. They prove that for every measurably k-colored d-dimensional necklace and for every nonnegative $t_1 + + t_d = k(q-1)$ there exists a fair splitting into q collections using only axis aligned hyperplane cuts and at most t_i cuts with hyperplanes perpendicular to i-th coordinate. We prove that this bound is sharp in a strong sense. Namely for every d-dimensional necklace there exists a measurable k-coloring such that for any $t_1 + + t_d = k(q-1) - 1$ there is no fair q-splitting using only axis aligned hyperplane cuts among which at most t_i hyperplanes are perpendicular to i-th coordinate. Our proof shows that the set of such colorings is dense. We also prove that there exists a measurable k-coloring of R^d such that no d-dimensional necklace has a fair q-splitting using at most k(q - 1) - d - 2 axis aligned hyperplane cuts, we conjecture that it is tight. This improves the result of Lubawski. For d = 1 we get the result of the paper of Alon, Grytczuk, Lason and Michalek. After them we use topological BaireŠs Theorem, but the rest of the argument relies on algebraic independence over suitably chosen fields. Moreover we study the case of arbitrary hyperplane cuts (not necessary axis aligned). Additionally we look at a discrete version, for higher dimensions it differs much from a continuous one. We give bounds on the minimal number of axis aligned hyperplane cuts t that are needed to fairly q-split any (discrete) d-dimensional k-colored necklace (a grid).

Allan Lo
University of Birmingham, UK

Properly coloured Hamiltonian cycles in edge-coloured K_n

Let K_n^c be an edge-coloured complete graph on n vertices. Let $\Delta_{\text{mon}}(K_n^c)$ denote the maximum number of edges of the same colour incident with a vertex of K_n^c. In other words, $\Delta_{\text{mon}}(K_n^c)$ is the largest maximum degree $\Delta(H)$ of H over all monochromatic subgraphs H of K_n^c. In 1976, Bollobás and Erdős conjectured that every K_n^c with $\Delta_{\text{mon}}(K_n^c) < \lfloor n/2 \rfloor$ contains a properly coloured Hamiltonian cycle, that is, a spanning cycle in which adjacent edges have distinct colours. Alon and Gutin showed that for any $\varepsilon > 0$ and $n \geq n_0(\varepsilon)$ if $\Delta_{\text{mon}}(K_n^c) < (1 - 1/\sqrt{2} - \varepsilon)n$, then K_n^c contains a properly coloured Hamiltonian cycle. In this talk, we show that $\Delta_{\text{mon}}(K_n^c) < (1/2 - \varepsilon)n$ is sufficient (again for any $\varepsilon > 0$ and $n \geq n_0(\varepsilon)$). Hence, the conjecture of Bollobás and Erdős is true asymptotically. If time permits, we will also discuss an analogue of Dirac's theorem for properly coloured Hamiltonian cycles in edge-coloured graphs.

Piotr Micek
Jagiellonian University, Poland

COLORING INTERSECTION GRAPHS OF CURVES AND ARC-WISE CONNECTED SETS IN THE PLANE

joint work with: Michał Lasoń[1], Arkadiusz Pawlik[1], Bartosz Walczak[1]
[1]Jagiellonian University, Poland

A class of graphs is χ-bounded if there is a function f such that for any graph G from the class $\chi(G) \leq f(\omega(G))$. In 2012, two long-standing open problems on χ-boundedness of geometric objects in the plane have been resolved: (1) the intersection graphs of line segments in the plane are not χ-bounded; (2) the intersection graphs of unit-length line segments in the plane are χ-bounded. In order to prove (2) Andrew Suk has shown that the intersection graphs of simple family of x-monotone curves all pierced by a common vertical line are χ-bounded. We improve the latter result getting rid of x-monotonicity restriction. Further improvements allow us to state that simple families of arc-wise connected, compact sets in the plane all pierced by a common line are χ-bounded, which generalizes the results of McGuinness and others.

Katarzyna Mieczkowska
Adam Mickiewicz University, Poland

On matchings in hypergraphs

joint work with: Tomasz Łuczak[1]
[1]Adam Mickiewicz University, Poland

In 1965 Erdos conjectured that the number of edges in k-uniform hypergraphs on n vertices in which the largest matching has s edges is maximized either for cliques, or for graphs which consist of all edges intersecting a set of s vertices. We settled this conjecture in the afirmative in the case in which $k = 3$ and n is large enough.

Mirjana Mikalački
University of Novi Sad, Serbia

Doubly biased Maker - Breaker Connectivity game

joint work with: Miloš Stojaković[1], Dan Hefetz[2]
[1]University of Novi Sad, Serbia, [2]University of Birmingham, UK

Let V be a finite set and let $\mathcal{F} \subseteq 2^V$ be the family of its subsets. Positional game is a pair $(V, \mathcal{F})$, where V is referred to as *board*, and the sets of $\mathcal{F}$ is referred to as the *winning sets*. In the $(a : b)$ *Maker - Breaker* game, a type of positional game, two players called *Maker* and *Breaker* take turns in claiming previously unclaimed elements of the board. Maker claims a elements in each move and *Breaker* claims b elements in each move. The game ends when all the elements of the board are claimed. Maker aims to claim all the elements of some $F \in \mathcal{F}$ and Breaker wants to prevent him from doing that, i.e. he wants to put at least one element in each winning set. The game is *"Maker's win"* if Maker has a strategy to win against any strategy of Breaker. Otherwise, the game is *"Breaker's win"*.

When $a = b = 1$ the game is called *unbiased*, and in many $(1 : 1)$ Maker - Breaker games, Maker wins quite easily. Thus, to "even out the odds" and increase Breaker's chances to win, the *biased* games are introduced. The question that comes naturally is to determine the winner of $(1 : b)$ Maker - Breaker game, when b is greater than one, and to increase b until the game becomes more balanced.

We study $(a : b)$ Maker - Breaker Connectivity game played on the edge set of the complete graph on n vertices, $E(K_n)$, where n is sufficiently large integer and both a and b can be greater than one. The winning sets we look at are spanning trees. For each $a = a(n)$, we determine $b_0(a, n) = b_0(a)$, so that for $b < b_0(a)$ the game is Maker's win, and for $b > b_0(a)$ the game is Breaker's win. We refer to $b_0(a)$ as the *threshold bias for a.*

Kevin Milans
University of Louisville, USA

Forbidden Induced Posets in the Boolean Lattice

joint work with: Linyuan Lu[1]
[1]University of South Carolina, USA

For a poset P, the Turán number $\mathrm{La}(n, P)$ is the maximum size of a family of elements in the n-dimensional Boolean lattice that does not contain P as a subposet. The asymptotics of the Turán number is known asymptotically for several families of posets, including the family of posets whose Hasse diagram is a tree [Bukh]. The induced Turán number $\mathrm{La}^*(n, P)$ is the maximum size of a family of elements in the n-dimensional Boolean lattice that does not contain P as *an induced* subposet. While the induced Turán number in general behaves quite differently from the Turán number, nonetheless Boehnlein and Jiang recently extended Bukh's result to show that the asymptotics of $\mathrm{La}^*(n, P)$ and $\mathrm{La}(n, P)$ are identical when the Hasse diagram of P is a tree. Much less is known about $\mathrm{La}^*(n, P)$ when the Hasse diagram of P contains cycles. We present bounds on $\mathrm{La}^*(n, P)$ when P is a series-parallel poset or the standard example. This is joint work with Linyuan Lu.

Lothar Narins
Freie Universität Berlin, Germany

Ryser's Conjecture and Home-Base Hypergraphs

joint work with: Penny Haxell[1], Tibor Szabó[2]
[1]University of Waterloo, Canada, [2]Freie Universität Berlin, Germany

Ryser's Conjecture states that any r-partite r-uniform hypergraph has a vertex cover of size at most r-1 times the size of the largest matching. For r=2, the conjecture is simply König's Theorem. It has also been proven for r=3 by Aharoni using topological methods. Our ambitious goal is to try to extend Aharoni's proof to r=4. We are currently still far from this goal, but we start by characterizing those hypergraphs which are tight for the conjecture for r=3. These we call home-base hypergraphs. Our proof of this characterization is also based on topological machinery, particularly utilizing results on the (topological) connectedness of the independence complex of the line graph of a graph.

Mateusz Nikodem
AGH University of Science and Technology, Poland

Property stability of graphs

joint work with: Sylwia Cichacz[1], Agnieszka Görlich[1], Andrzej Żak[1],
[1]AGH University of Science and Technology, Poland

Let P be some graph property and $\mathcal{H}$ be the set of all graphs with property P. We say that graph G is $(H;k)$-stable if after removing any k of its vertices, the remaining graph contains some representative of H as a subgraph. We are interested in a problem of finding $(H;k)$-stable graphs of minimal size. This is a generalization of the problem of vertex stability of graphs regarding just to one fixed graph (instead of set graphs).

In this talk we present some results regarding to $H = H(n)$ being a set of all connected graphs of order n or more.

Liudmila Ostroumova
Moscow State University, Russia

The distribution of second degrees in preferential attachment models

joint work with: Evgeniy Grechnikov[1], Andrei Kupavskii[2], Prasad Tetali[3]
[1]Bauman Moscow State Technical University, Russia, [2]Moscow State University, Russia
[3]Georgia Institute of Technology, USA

We consider some properties of preferential attachment random graph models. The first model was suggested by Barabási and Albert and defined more precisely by Bollobás and Riordan. At each step we add one vertex and one edge. The probability that a new vertex will be connected to some previous vertex v is proportional to the degree of v. Bollobás and Riordan proved that in this model the degree sequence has a power-law distribution.

The second model was explicitly defined by Buckley and Osthus. This model is a generalization of the previous one and has an additional parameter - an "initial attractiveness" of a node which is a positive constant that does not depend on the degree. In this model the degree sequence also follows a power law distribution. The parameter of this distribution depends on initial attractiveness of vertices.

We study the second degrees of vertices in preferential attachment models. Roughly speaking, the second degree of a vertex is the number of vertices at distance two from this vertex. The distribution of second degrees is of interest because it is a good approximation of PageRank, where the importance of a vertex is measured by taking into account the popularity of its neighbors.

We prove that the second degrees in preferential attachment models also obey a power law. We estimate the expectation of the number of vertices with the second degree k and prove the concentration of this random variable around its expectation. We compare results obtained by using Azuma's inequality and Talagrand's concentration inequality over product spaces. As far as we know this is the only application of Talagrand's inequality to random web graphs, where the (preferential attachment) edges are not defined over a product distribution, making the application nontrivial, and requiring certain novelty.

Balazs Patkos
Alfred Renyi Institute of Mathematics, Hungary

On traces of set families

An old result of Erdős states that if a family $\mathcal{F} \subseteq 2^{[n]}$ does not contain a chain of length $k+1$ (i.e. sets $F_1 \subsetneq F_2 \subsetneq \ldots \subsetneq F_k \subsetneq F_{k+1}$), then its size cannot be larger than $\sum_{i=1}^{k} \lfloor \frac{n-k}{2} + i \rfloor$. Another topic in extremal finite set theory deals with problems concerning *traces* of set families. The trace of a set F on another set X is $F|_X = F \cap X$, while the trace of a family $\mathcal{F}$ is $\mathcal{F}|_X = \{F|_X : F \in \mathcal{F}\}$. The fundamental theorem about traces is the so-called Sauer-lemma (proved independently by Sauer, Shelah, and Vapnik and Chervonenkis) that states that if $\mathcal{F} \subseteq 2^{[n]}$ contains more than $\sum_{i=0}^{l-1} \binom{n}{i}$ sets, then there exist an $L \in \binom{[n]}{l}$ such that $\mathcal{F}|_L = 2^L$.

A family of sets $\mathcal{F} \subseteq 2^{[n]}$ is defined to be l-trace k-Sperner if for any l-subset L of $[n]$ the family of traces $\mathcal{F}|_L$ does not contain any chain of length $k+1$. After summarizing some earlier results, we shall prove that for any positive integers l', k with $l' < k$ if $\mathcal{F}$ is $(n-l')$-trace k-Sperner, then $|\mathcal{F}| \leq (k - l' + o(1))\binom{n}{\lfloor n/2 \rfloor}$ and this bound is asymptotically tight. Moreover, for any $k \geq 1$ we shall prove that the maximum size that an $(n-1)$-trace $(k+1)$-Sperner family $\mathcal{F} \subseteq 2^{[n]}$ can have is $\sum_{i=1}^{k} \lfloor \frac{n-k}{2} + i \rfloor$ and we shall characterize the extremal families.

Arkadiusz Pawlik
Jagiellonian University, Poland

Triangle-free intersection graphs of line segments with large chromatic number

joint work with: Bartosz Walczak[1], Jakub Kozik[1], Tomasz Krawczyk[1], Michał Lasoń[1] Piotr Micek[1], William Trotter [2]
[1]Jagiellonian University, Poland , [2]Georgia Institute of Technology, USA

In the 1970s Erdős asked whether the chromatic number of intersection graphs of line segments in the plane is bounded in terms of the size of the largest clique. We show the answer is no. Specifically, for each positive integer k we construct a triangle-free family $\mathcal{S}_k$ of line segments in the plane with chromatic number greater than k. Furthermore, the very same graphs can be represented by intersections of other planar objects like axis-alligned rectangular frames or L-shapes. The latter provides a negative answer to a question of Gyárfás and Lehel (1985). Our construction also disproves a purely graph-theoretical conjecture of Scott (1997).

Guillem Perarnau
Universitat Politécnica de Catalunya, Spain

Matchings in Random Biregular Bipartite Graphs

joint work with: Giorgis Petridis[1],
[1]University of Cambridge, UK

We study the existence of perfect matchings in suitably chosen induced subgraphs of random biregular bipartite graphs. We prove a result similar to a classical theorem of Erdos and Renyi about perfect matchings in random bipartite graphs. We also present an application to commutative graphs, a class of graphs that are featured in additive number theory.

Monika Pilśniak
AGH University of Science and Technology, Poland

Endomorphism Breaking in Graphs

joint work with: Wilfried Imrich[1], Rafał Kalinowski[2]
[1]Montanuniversität Leoben, Austria, [2]AGH University of Science and Technology, Poland

We introduce the *endomorphism distinguishing number* $D_e(G)$ of a graph G as the least cardinal d such that G has a vertex coloring with d colors that is only preserved by the trivial endomorphism. This is a natural generalization of the distinguishing number $D(G)$ of a graph G, which is defined for automorphisms instead of endomorphisms.

As the number of endomorphisms usually vastly exceeds the number of automorphisms, the new concept opens challenging problems, several of which are presented here. Moreover, there are numerous results about the distinguishing number of finite and infinite graphs that can be extended to the endomorphisms distinguishing number. This is the main concern of the talk.

Leonid Plachta
AGH University of Science and Technology, Poland

Singular links and semicoloured planar bipartite graphs

In this talk, we discuss some known problems in the theory of singular links and indicate how they can be interpreted in pure combinatorial terms, i.e. semicoloured planar bipartite graphs. We also show how structural properties of semiloloured graphs reflect some topological properties of singular links and their diagrams.

Ekaterina Ponomarenko
Moscow State University, Russia

On Borsuk's and Nelson–Hadwiger's problems for rational spaces

joint work with: A.B. Kupavskii[1], A.M. Raigorodskii[1],
[1]Moscow State University, Russia

In 1933 Borsuk conjectured that every non-singleton bounded point set in $\mathbb{R}^n$ can be partitioned into $n+1$ parts of smaller diameter. Borsuk's conjecture was disproved only in 1993. Now, the quantity $f(n)$ is considered, which is the minimum number such that any bounded non-singleton set in $\mathbb{R}^n$ can be partitioned into $f(n)$ parts of smaller diameter. In particular, it is known that $f(n) = n+1$ for $n \le 3$; $f(n) > n+1$ for $n \ge 298$;

$$\left(\left(\frac{2}{\sqrt{3}}\right)^{\sqrt{2}} + o(1)\right)^{\sqrt{n}} \le f(n) \le \left(\sqrt{\frac{3}{2}} + o(1)\right)^n.$$

In our talk, we shall discuss some variants of the problem in which $\mathbb{R}^n$ is substituted by $\mathbb{Q}^n$ that consists of all rational vectors in the Euclidean space. Namely, let $V \subset \mathbb{Q}^n$, diam $V = 1$. Denote by $\chi_{\mathbb{Q}}(V)$ the minimum number of colors needed to color V so that any two monochromatic points in V are not at the distance 1 apart. Let $\chi_{\mathbb{Q},1}(n) = \max_V \chi_{\mathbb{Q}}(V)$. The quantity $\chi_{\mathbb{Q},1}(n)$ is a good rational analog of the Borsuk number $f(n)$.

Further, by the *affine dimension* affdim V of a set $V \subset \mathbb{R}^n$ we mean the minimum dimension of an affine subspace $H \subset \mathbb{R}^n$ that contains V. Another rational analog of the Borsuk number is

$$\chi_{\mathbb{Q},1}^{\mathrm{aff}}(n) = \max_{V:\ \mathrm{affdim\,V=n,\ diam\,V=1}} \chi_{\mathbb{Q}}(V).$$

The first part of our talk will be devoted to various bounds for the values $\chi_{\mathbb{Q},1}(n)$ and $\chi_{\mathbb{Q},1}^{\mathrm{aff}}(n)$.

In the second part of the talk, we shall speak about similar questions concerning the chromatic numbers of spaces. For example, we shall study the value $\chi(\mathbb{Q}^n)$, which is the minimum number of colors needed to color all the rational vectors in the Euclidean space so that there are no pairs of monochromatic points at a given rational distance apart. We shall also study some generalizations.

Jakub Przybyło
AGH University of Science and Technology, Poland

Challenges and landmarks in vertex distinguishing graph colourings

A vertex colouring of a graph is proper if it distinguishes adjacent vertices, i.e., attributes distinct colours to ends of every edge. The minimum number of colours required is then known as the chromatic number of the graph. Suppose now that we aim at distinguishing (adjacent) vertices by assigning colours to edges instead. A variety of problems concerns determining the minimum number of colours necessary in edge colourings distnguishing adjacent or all vertices of a graph e.g. by the pallets of colours of their incident edges. Furthermore, we might also colour the edges with numbers and require the sums of these to be distinct in every pallet. Such additive variations, which in fact motivated the others, are usually more challenging. In the talk we shall present a range of interesting problems in this area, which are almost all open, such as $1-2-3$-Conjecture or a daring conjecture by Zhang Liu and Wang. These shall be supplemented with most recent partial results and examples of different approaches towards solving them. Many of these approaches provide algorithms for constructing the corresponding colourings, enhanced e.g. by probabilistic tools or an application of Combinatorial Nullstellensatz.

Michał Jan Przykucki
University of Cambridge, United Kingdom

Maximum percolation time in bootstrap percolation

joint work with: Fabricio Benevides[1]
[1]Universidade Federal do Ceará , Brasil

Bootstrap percolation is one of the simplest cellular automata. In r-neighbour bootstrap percolation on a graph G an infection spreads according to the following deterministic rule: infected vertices of G remain infected forever and in consecutive rounds healthy vertices with at least r already infected neighbours become infected. Percolation occurs if eventually every vertex is infected.

In this talk we focus on extremal problems in 2-neighbour bootstrap percolation and present our recent results about the maximum time this process can take to percolate in the $n \times n$ square grid and in the n-dimensional hypercube graph.

Andrei Raigorodskii
Moscow State University, Russia

On the chromatic numbers of spaces with sets of forbidden equilateral triangles

joint work with: D.V. Samirov[1]
[1]Moscow State University, Russia

In this work, we deal with a generalization of the classical chromatic number of $\mathbb{R}^n$. By *the chromatic number* of $\mathbb{R}^n$ we mean the quantity $\chi(\mathbb{R}^n)$ equal to the minimum number of colors needed to color all the points in $\mathbb{R}^n$ so that any two points at the distance 1 apart receive different colors. The problem of finding the value of $\chi(\mathbb{R}^n)$ goes back to Nelson, Erdős, and Hadwiger. Many results concerning it have been obtained. For example, we know that $\chi(\mathbb{R}^n)$ grows exponentially as n tends to infinity. However, the exact value of the chromatic number is not found even in the case of $\mathbb{R}^2$.

Here, instead of forbidding monochromatic pairs of points, we forbid monochromatic triples of points ("triangles"). More precisely, we take arbitrary numbers a, b such that $0 < a < 1 < b < 2$. We consider the quantity $\chi_{a,b}(\mathbb{R}^n)$ which is the minimum number of colors needed to color all the points in $\mathbb{R}^n$ so that no three monochromatic points are the vertices of an equilateral triangle whose equal sides are of length 1 and the third side is of length between a and b.

In our talk, we shall give a survey of the problems around the chromatic numbers of spaces. Then we shall present our new results concerning the quantities $\chi_{a,b}(\mathbb{R}^n)$.

Katarzyna Rybarczyk
Adam Mickiewicz University, Poland

Independent sets in a random intersection graph

joint work with: Konstanty Junosza-Szaniawski[1], Jan Kratochvil[2], Martin Pergel[2]
[1]Warsaw University of Technology, Poland, [2]Charles University, Prague, Czech Republic

In the random intersection graph model $G(n, m, P_{(m)})$ to each vertex from a vertex set V ($|V| = n$) we assign, independently from all other vertices, a random set of its features $W(v)$ from an auxiliary set W ($|W| = m$). The cardinality of $W(v)$ is chosen according to a given probability distribution $P_{(m)}$ and $W(v)$ is chosen uniformly at random from all subsets of W of this cardinality. We connect vertices v and u by an edge if the sets $W(v)$ and $W(u)$ intersect. Due to large edge dependency, for a wide range of parameters $G(n, m, P_{(m)})$ exhibit different behavior than that of an Erdős Rényi random graph $G(n, p)$ with independent edges. We will present new results concerning the independence number of $G(n, m, P_{(m)})$ in two most studied cases, for $P_{(m)}$ – the distribution with point mass in $d(n)$ and $P_{(m)}$ – the binomial distribution $\mathrm{Bin}(\mathrm{m}, \mathrm{p})$. In both cases we determine the asymptotic value of the independence number. Moreover we provide the detailed analysis of the greedy algorithm on those graphs. We also propose a new greedy algorithm, which rely heavily on the structure of a random intersection graph and in many cases performs far better than the classical one. As the conclusion we determine the range of parameters for which greedy algorithms give better results for $G(n, m, P_{(m)})$ than this is in the case of an Erdős Rényi random graph.

Paweł Rzążewski
Warsaw University of Technology, Poland

Beyond homothetic polygons: recognition and maximum clique

joint work with: Konstanty Junosza-Szaniawski[1], Jan Kratochvil[2], Martin Pergel[2]
[1]Warsaw University of Technology, Poland, [2]Charles University, Prague, Czech Republic

We study the Clique problem in classes of intersection graphs of convex sets in the plane. The problem is known to be NP-complete in convex-sets intersection graphs and straight-line-segments intersection graphs, but solvable in polynomial time in intersection graphs of homothetic triangles. We extend the latter result by showing that for every convex polygon P with k sides, every n-vertex graph which is an intersection graph of homothetic copies of P contains at most n^{2k} inclusion-wise maximal cliques. We actually prove this result for a more general class of graphs, so called k_{DIR}-CONV, which are intersection graphs of convex polygons whose all sides are parallel to at most k directions. We further provide lower bounds on the numbers of maximal cliques, discuss the complexity of recognizing these classes of graphs and present relationship with other classes of convex-sets intersection graphs.

Dmitri Samirov
Moscow State University, Russia

Coloring spaces without forbidden configurations

joint work with: A.A. Harlamova[1], A.M. Raigorodskii[1], A.E. Zvonarev[1]
[1]Moscow State University , Russia

In 1950 E. Nelson proposed to find the quantity

$$\chi(\mathbb{R}^n) = \min\{\chi : \ \mathbb{R}^n = V_1 \sqcup \ldots \sqcup V_\chi \ \ \forall\, i \ \ \forall\, \mathbf{x}, \mathbf{y} \in V_i \ \ |\mathbf{x} - \mathbf{y}| \neq 1\}.$$

During the last 60 years, a lot of results concerning $\chi(\mathbb{R}^n)$ have been obtained. In particular, as $n \to \infty$, we have

$$(1.239\ldots + o(1))^n \leq \chi(\mathbb{R}^n) \leq (3 + o(1))^n.$$

Here the lower bound is due to A.M. Raigorodskii and the upper bound belongs to D.G. Larman and C.A. Rogers.

Moreover, some important generalizations of the above-described quantity have been proposed. The so-called Euclidean Ramsey theory deals in fact with the following situation. Let $\mathcal{A}$ be a set of points in a space $\mathbb{R}^d$. Let $V \subset \mathbb{R}^n$. We say that $\mathcal{A}$ *appears* in V, provided there exists a point configuration in V that is congruent to $\mathcal{A}$. Consider

$$\chi(\mathbb{R}^n; \mathcal{A}) = \min\{\chi : \ \mathbb{R}^n = V_1 \sqcup \ldots \sqcup V_\chi \ \ \forall\, i \ \mathcal{A} \text{ does not appear in } V_i\}.$$

In the Euclidean Ramsey theory a different terminology is used. For instance, it is actually proved that for many configurations $\mathcal{A}$, the value of $\chi(\mathbb{R}^n; \mathcal{A})$ is exponential in n, and in this case, the set $\mathcal{A}$ is called *exponentially Ramsey*. Any simplices are among exponentially Ramsey sets. However, no explicit lower bounds for $\chi(\mathbb{R}^n; \mathcal{A})$ have been obtained. In our work, we find a series of explicit exponential lower bounds for $\chi(\mathbb{R}^n; \mathcal{A})$, where $\mathcal{A}$ is the vertex set of a triangle, thus generalizing the estimate

$$\chi(\mathbb{R}^n) = \chi(\mathbb{R}^n; \{1, 2\}) \geq (1.239\ldots + o(1))^n.$$

Our technique can be applied for many other configurations.

Marko Savić
University of Novi Sad, Serbia

Linear Time Algorithm for Optimal Feed-link Placement

joint work with: Miloš Stojaković[1]
[1]University of Novi Sad , Serbia

A polygon representing transportation network is given, together with a point p in its interior. We aim to extend the network by inserting a line segment, called a feed-link, which connects p to the boundary of the polygon. Geometric dilation of some point q on the boundary is the ratio between the length of the shortest path from p to q through the extended network and their Euclidean distance. The utility of a feed-link is inversely proportional to the maximal dilation over all boundary points. We give a linear time algorithm for computing the feed-link with the minimum overall dilation, thus improving upon the previously known algorithm of complexity close to $O(n \log n)$.

Dmitry Shabanov
Moscow State University, Russia

Colorings of uniform hypergraphs with large girth

joint work with: Andrey Kupavskii[1]
[1]Moscow State University, Russia

The work deals with a combinatorial problem concerning colorings of uniform hypergraphs with large girth. In 1973 P. Erdős and L. Lovász established the following quantitative connection between the chromatic number of an n-uniform hypergraph H and its maximum vertex degree $\Delta(H)$: if H is non-r-colorable then

$$\Delta(H) \geqslant \frac{r^{n-1}}{e\,n}.$$

Several papers are devoted to the improvement of this classical theorem for different classes of uniform hypergraphs.

We give a new lower bound for the maximum vertex degree in an n-uniform hypergraph with girth at least 6 and high chromatic number. Let $H = (V, E)$ be a hypergraph. A *simple cycle of length k* in H is a sequence $e_0, v_0, e_1, v_1, \ldots, e_{k-1}, v_{k-1}, e_k = e_0$ of k distinct edges $e_0, \ldots, e_{k-1}$ and k distinct vertices $v_0, \ldots, v_{k-1}$ such that $v_i \in e_i \cap e_{i+1}$ for all $i = 0, \ldots, k-1$. The length of the shortest simple cycle in H is called *the girth* of the hypergraph. Our main result is formulated in Theorem 1.

Theorem 1. *Let $n \geqslant 3$ and $r \geqslant 2$. There exists an absolute constant $c > 0$ such that, for any n-uniform non-r-colorable hypergraph H with girth at least 6, the following inequality holds*

$$\Delta(H) \geqslant c\,\frac{r^{n-1}}{\ln n}.$$

For example, $c = 1/110$ is sufficient.

The proof of Theorem 1 is based on the random recoloring method and uses a continuous-time random recoloring process.

Joanna Skowronek-Kaziów
University of Zielona Góra, Poland

Multiplicative Vertex-Colouring Weightings of Graphs

It is conjectured that the edges of any non-trivial graph can be weighted with integers $1, 2, 3$ in such a way that for every edge uv the product of weights of the edges adjacent to u is different than the product of weights of the edges adjacent to v. The author proves it for cycles, paths, complete graphs and 3-colourable graphs. It is also shown that the edges of every non-trivial graph can be weighted with integers $1, 2, 3, 4$ in such a way that the adjacent vertices have different products of incident edge weights. In a total weighting of a simple graph G we assign the positive integers to edges and to vertices of G. We consider a colouring of G obtained by assigning to each vertex v the product of its weight and the weights of its adjacent edges. It is conjectured that we can get the proper colouring in this way using the weights $1, 2$ for every simple graph. It is shown that we can do it using the weights $1, 2, 4$ on edges and $1, 2$ on vertices.

Katherine Staden
University of Birmingham, UK

Approximate Hamilton decompositions of regular expanders

joint work with: Deryk Osthus[1],
[1]Birmingham University, UK

We show that every sufficiently large r-regular digraph G which has linear degree and is a robust outexpander has an approximate decomposition into edge-disjoint Hamilton cycles, i.e. G contains a set of r-o(r) edge-disjoint Hamilton cycles. Here G is a robust outexpander if for every set S which is not too small and not too large, the 'robust' outneighbourhood of S is a little larger than S. This generalises a result of Kühn, Osthus and Treglown on approximate Hamilton decompositions of dense regular oriented graphs. It also generalises a result of Frieze and Krivelevich on approximate Hamilton decompositions of quasirandom (di)graphs. In turn, our result is used as a tool by Kühn and Osthus to prove that any sufficiently large r-regular digraph G which has linear degree and is a robust outexpander even has a Hamilton decomposition.

Miloš Stojaković
University of Novi Sad, Serbia

Winning fast in positional games on graphs

For various standard positional games played on the edges of the complete graph for which it is clear who the winner is, we try to determine how fast can that player win.

Małgorzata Sułkowska
Wrocław University of Technology, Poland

The best choice problem for upward directed graphs

We consider a generalization of the best choice problem to upward directed graphs. We describe a strategy for choosing a maximal element (i.e., an element with no outgoing edges) when a selector knows in advance only the number n of vertices of the graph. We show that, as long as the number of elements dominated directly by the maximal ones is not greater than $c_1\sqrt{n}$ for some positive constant c_1 and the indegree of remaining vertices is bounded by a constant D, the probability p_n of the right choice according to our strategy satisfies $\liminf_{n\to\infty} p_n\sqrt{n} \geq \delta > 0$, where δ is a constant depending on c_1 and D (which is best possible up to a constant).

Paulius Šarka
Vilnius University, Lithuannia

Large co-Sidon subsets in sets with a given additive energy

joint work with: Michitaka Furuya[1],
[1]Tokyo University of Science, Japan

For two finite sets of integers A and B their additive energy $\mathsf{E}(A,B)$ is the number of solutions to $a+b=a'+b'$, where $a,a'\in A$ and $b,b'\in B$. Given finite sets $A,B\subseteq \mathbf{Z}$ with additive energy $\mathsf{E}(A,B)=|A||B|+E$, we investigate the sizes of largest subsets $A'\subseteq A$ and $B'\subseteq B$ with all $|A'||B'|$ sums $a+b$, $a\in A', b\in B'$, being different (we call such subsets A',B' *co-Sidon*).

In particular, for $|A|=|B|=n$ we show that in the case of small energy, $n\le E=\mathsf{E}(A,B)-|A||B|\ll n^2$, one can always find two co-Sidon subsets A',B' with sizes $|A'|=k, |B'|=\ell$, whenever k,ℓ satisfy $k\ell^2\ll n^4/E$. We also give an example, showing that this is best possible up to the logarithmic factor.

When the energy is large, $E\gg n^3$, we show that there exist co-Sidon subsets A',B' of A,B with sizes $|A'|=k, |B'|=\ell$ whenever k,ℓ satisfy $k\ell\ll n$ and show that this is best possible. We will also discuss extensions of these results to the full range of values of E.

Shoichi Tsuchiya
Tokyo University of Science, Japan

Forbidden pairs for the existence of a homeomorphically irreducible spannig tree

joint work with: Michitaka Furuya[1],
[1]Tokyo University of Science, Japan

A spanning tree with no vertices of degree two of a graph is called a *homeomorphically irreducible spanning tree* (or *HIST*) of the graph. Let $\mathcal{H}$ be a family of connected graphs. A graph G is said to be $\mathcal{H}$*-free* if G is has no H as an induced subgraph for every graph H in $\mathcal{H}$. We also say that the members of $\mathcal{H}$ are *forbidden subgraphs*. It has been characterized sets of forbidden subgraphs that imply the existence of a HIST in a connected graph of sufficiently large order. In this talk, we characterize forbidden pairs (i.e., exactly two forbidden subgraphs) that imply the existence of a HIST in connected graphs. Moreover, we also characterize such pairs for the existence of a HIST in 2-connected graphs.

Zsolt Tuza
Renyi Institute, Hungary

Turan numbers and batch codes

joint work with: Csilla Bujtas[1],
[1]University of Veszprem, Hungary

We study hypergraph Turan numbers of the following kind: Given n, r, k, s (integers at least 2), determine the largest number of edges in an r-uniform hypergraph on n vertices, such that every s-element set of vertices contains fewer than k edges. Following the works of Brown, Erdos and T. Sos, mainly the case $k < s$ was studied. We consider the other range, $k > s$, which recently became motivated by its relation to "combinatorial batch codes", too. Such codes were introduced in the field of distributed information storage and retrieval. Beside the Turan-type results, in the talk we also give a briev overview of combinatorial (extremal) results concerning batch codes.

Maciej Ulas
Jagiellonian University, Poland

Arithmetic properties of hyper k-ary partition function

In the talk we present several interesting properties of hyper k-ary partition function $f_k(n)$ which counts the number of representation of an integer n in the form $n = \sum_{i=0}^{m} a_i k^i$, where $a_i \in \{0, 1, \ldots, k\}$ for $i = 0, 1, \ldots, m$. This sequence is a natural generalization of classical Stern diatomic sequence.

Andrew Uzzell
Uppsala Universitet, Sweden

The Speed of Bootstrap Percolation

joint work with: B. Bollobás[1], C. Holmgren[2], and P. J. Smith[1],
[1]Univerity of Mamphis, USA, [2]Uppsala University, Sweden

In r-neighbor bootstrap percolation on the vertex set of a graph G, vertices are initially infected independently with some probability p. At each time step, the infected set expands by infecting all uninfected vertices that have at least r infected neighbors. We study the distribution of the time at which all vertices become infected. Given $d \geq r \geq 2$ and $t = t(n) = o((\log n / \log \log n)^{1/(d-r+1)})$, we prove a sharp threshold result for the probability that percolation occurs by time t in r-neighbor bootstrap percolation on the d-dimensional discrete torus T_n^d. Moreover, we show that for certain ranges of $p = p(n)$, the time at which percolation occurs is concentrated either on a single value or on two consecutive values. We also prove corresponding results for the modified d-neighbor rule and for the subcritical case $d + 1 \leq r \leq 2d$.

Bartosz Walczak
Jagiellonian University, Poland

Outerplanar graph drawings with few slopes

joint work with: Kolja Knauer[1], Piotr Micek[2]
[1]Technische Universität Berlin, Germany, [2]Jagiellonian University, Poland

The slope number of a graph G is the minimum number s such that G has a drawing in the plane with edges represented by straight-line segments parallel to s disctinct directions. The slope number of a graph can be arbitrarily large even if its maximum degree is bounded by 5. On the other hand, the slope number is bounded for all planar graphs by a function of maximum degree, even when the drawing is additionally required to be planar.

We consider outerplanar straight-line drawings of outerplanar graphs and provide a tight bound for their slope number in terms of maximum degree. Specifically, we prove that $\Delta - 1$ slopes suffice for every outerplanar graph with maximum degree $\Delta \geq 4$. This improves on the previous bound of $O(\Delta^5)$, which was shown for planar partial 3-trees, a superclass of outerplanar graphs. On the other hand, for every $\Delta \geq 4$ there is an outerplanar graph of maximum degree Δ which requires at least $\Delta - 1$ distinct edge slopes for an outerplanar straight-line drawing.

Douglas West
Zhejiang Normal University and University of Illinois, USA

1, 2, 3-Conjecture and 1, 2-Conjecture for graphs with maximum average degree less than 8/3

joint work with: Daniel Cranston[1], Sogol Jahanbekam[2]
[1]Virginia Commonwealth University, USA, [2]University of Illinois, USA

The 1, 2, 3-Conjecture states that the edges of a graph with no isolated edge can be labeled from $\{1, 2, 3\}$ so that at adjacent vertices the totals of the incident labels are distinct. The 1, 2-Conjecture states that the edges and vertices can be labeled using $\{1, 2\}$ so that at adjacent vertices the totals of (incident labels plus vertex label) are distinct. We apply the Discharging Method to prove the 1, 2, 3-Conjecture and the 1, 2-Conjecture for graphs with maximum average degree less than 5/2. As a result, the conjectures hold for planar graphs with girth at least 10. The same method with considerably more effort will prove the results for maximum average degree less than 8/3 (including planar graphs with girth at least 8.

Krzysztof Węsek
Warsaw University of Technology, Poland

b-Bounded circular coloring of graphs

r-Circular coloring of a graph G is an assignment of open unit-length arcs on an Euclidean circle of length r, such that adjacent vertices get disjoint arcs. This concept can be useful to model optimization problems when we are considering tasks performed in the continuous, cyclic system and we are looking for the best schedule of tasks.

In this talk we introduce its natural modification, b-bounded circular coloring, motivated by the practical applications. It rises by adding of an additional condition to the basic circular coloring model: constraint corresponding to the restriction that (because of some technical limitations) we can perform at most b tasks simultaneously. Namely, we require that no point of the circle (in our interpretation: a time-point in the schedule) is contained by more than b arcs. b-Bounded circular chromatic number of a graph G can be defined as:

$$\chi_c^{b\text{-}bnd}(G) = \inf\{r : G \text{ has a } b\text{-bounded } r\text{-circular coloring}\}$$

We present some more or less fundamental properties of $\chi_c^{b\text{-}bnd}$.

Stephen Young
University of Louisville, USA

A Alon-Boppana result for the normalized Laplacian

We prove a Alon-Boppana style bound for the spectral gap of the normalized Laplacian for general graphs via a bound on the weighted spectral radius of the universal cover graph.

Maksim Zhukovskii
Moscow State University, Russia

On the convergence of probabilities of first-order properties when the zero-one k-law for random graphs doesn't hold

In this talk, we shall discuss questions concerning the convergence of probabilities of first-order properties for random graphs in the Erdős–Rényi model $G(n,p)$. It was proved in 1969 by Y.V. Glebskii, D.I. Kogan, M.I. Liagonkii and V.A. Talanov that a probability of any first-order property tends to 0 or 1 if $p = \text{const}$. The same is true when p is a function of n such that $\min\{p, 1-p\}n^{\alpha} \to \infty$ as $n \to \infty$ for any $\alpha > 0$. When $p(n)$ is chosen in such a way that for any first-order property its probability tends to 0 or 1 it is said to *satisfy the zero-one law.* In 1988 S. Shelah and J.H. Spencer proved the following zero-one law.

Theorem 1. *Let $p = n^{-\alpha}$, where $\alpha \in (0,1)$ is an irrational number. Then the random graph $G(n,p)$ satisfies the zero-one law.*

When α is rational the zero-one law doesn't hold. In 2010 we considered the class $\mathcal{L}_k$ of all first-order properties expressed by formulas with quantifier depth bounded by a number k. We say that a function $p(n)$ (or the random graph $G(n,p)$) *satisfies the zero-one k-law* if for any property $L \in \mathcal{L}_k$ either $\lim_{n\to\infty} \mathsf{P}(L) \to 0$ or $\lim_{n\to\infty} \mathsf{P}(L) \to 1$. We proved the following fact.

Theorem 2. *Let $p = n^{-\alpha}$ and $k \geq 3$. If $0 < \alpha < 1/(k-2)$ then $G(n,p)$ satisfies the zero-one k-law.*

If $\alpha = 1/(k-2)$ the zero-one k-law doesn't hold. Recently we obtained the following convergence law.

Theorem 3. *Let $p = n^{-1/(k-2)}$ and $k \geq 3$. Then for any property $L \in \mathcal{L}_k$ there exists the limit $\lim\limits_{n\to\infty} \mathsf{P}(L)$.*

The proof of Theorem 3 is based on non-trivial geometric constructions and on a generalization of the graph extensions studied in 1990 by J.H. Spencer. In the talk, we shall discuss our approach.

Andrzej Żak
AGH University of Science and Technology, Poland

Hamilton saturated hypergraphs of essentially minimum size

joint work with: Andrzej Ruciński[1]
[1]Adam Mickiewicz University, Poland

For $1 \leq \ell < k$, an ℓ-overlapping cycle is a k-uniform hypergraph in which, for some cyclic vertex ordering, every edge consists of k consecutive vertices and every two consecutive edges share exactly ℓ vertices. A k-uniform hypergraph H is ℓ-Hamiltonian saturated, $1 \leq \ell \leq k-1$, if H does not contain an ℓ-overlapping Hamiltonian cycle $C_n^{(k)}(\ell)$ but every hypergraph obtained from H by adding one more edge does contain $C_n^{(k)}(\ell)$. Let $sat(n,k,\ell)$ be the smallest number of edges in an ℓ-Hamiltonian saturated k-uniform hypergraph on n vertices. Clark and Entringer proved in 1983 that $sat(n,2,1) = \lceil \frac{3n}{2} \rceil$. In this talk we will prove that $sat(n,k,\ell) = \Theta(n^{\ell})$ for $\ell = 1$ as well as for all $k \geq 5$ and $\ell \geq 0.8k$.